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The "more general" relation between mass, energy and momentum in special relativity

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In Newtonian mechanics, it is known that a mass m has rest energy $E = mc^2$ and momentum $p = mv$. This knowledge is to be extended to special relativity.

The following statement can be found in the literature on this topic:

> In special relativity, the relationship between energy (E), momentum (p) and mass (m) is described by the more general formula of the energy-momentum relation:
>
> $$E^2 = (mc^2)^2 + (pc)^2. \quad (1)$$

What does this equation state and how is it derived? A derivation can be found that proceeds as follows:

> In a stationary inertial frame of reference is
>
> $$E = m \cdot c^2. \quad (2)$$
>
> In a moving inertial frame of reference is
>
> $$m' = m \cdot \gamma \;, \qquad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \text{ – Lorentz transformation.} \quad (3)$$

This means that the rest mass m in the moving system increases by an amount that depends on its velocity. If we correctly denote the rest mass as m_0, then the total mass m can be understood as $m = m_0 + m_{dyn}$. The mass m increases by the amount of the dynamic (moving) mass in the moving inertial frame. This dynamic mass is negligibly small at velocities $v \ll c$, but at high velocities v near the speed of light, it is non-zero: $m_{dyn} > 0$.

Continuing the derivation:

> This results in the following for the moving system
>
> **a) The energy:**
>
> $$E = m \cdot \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \cdot c^2 \text{ and further} \quad (4)$$
>
> $$E^2 = m^2 \cdot c^4 \cdot \frac{1}{1-\frac{v^2}{c^2}}. \quad (5)$$
>
> **b) The momentum:**
>
> $$p = m \cdot v \cdot \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{and further} \quad (6)$$
>
> $$p^2 = (mv)^2 \cdot \frac{1}{1-\frac{v^2}{c^2}} \quad (7)$$
>
> $$(pc)^2 = (mv)^2 \cdot \frac{c^2}{1-\frac{v^2}{c^2}} \quad (8)$$
>
> Subtracting b) from a), results

$$E^2 - (pc)^2 = (m^2c^4 - m^2v^2c^2) \cdot \frac{1}{1-\frac{v^2}{c^2}} \text{ and further} \tag{9}$$

$$E^2 - (pc)^2 = (m^2c^4 - m^2v^2c^2) \cdot \frac{c^2}{c^2-v^2}$$

$$E^2 - p^2c^2 = m^2c^2 \cdot (c^2 - v^2) \cdot \frac{c^2}{c^2-v^2} = m^2c^4. \tag{10}$$

The reason for subtracting (8) from (5) at this point is not explained. This derivation step is suspicious. It cannot be explained physically. Ultimately, this results in:

$$\boldsymbol{E^2 = (mc^2)^2 + (pc)^2} \tag{11}$$

This subtraction, without justification, introduces the rest mass m instead of the moving mass $m‘$ into the calculation for the moving system, because the Lorentz factor appears to be eliminated in this process. Ultimately, this leads to the momentum being removed from the further calculation, as the verification below demonstrates.

The thesis currently officially held in physics is:

> For massless particles (particles without rest mass, meaning $m = 0$), momentum arises from moving energy. This simplifies the formula to:
> $E = p \cdot c$ oder $p = E/c$. (12)

This assumption arises from the premise that momentum p is also > 0 if $m = 0$. This is the error inherent in this approach. However one attempts to explain momentum, it is always $p = m' \cdot v$, and for $m = 0$ is also $m' = 0$ and therefore $p = 0$. Equation (12) consequently arose by setting the mass to zero in the first term of (11), but not in the second.

Verification:

Equation (11) yields:

$$E^2 = c^2 \cdot ((mc)^2 + p^2).$$

Instead of the impulse, I set $p = mv\gamma = m'v$, as stated in (6). This leads to

$$\left(\frac{E}{c}\right)^2 = m^2c^2 + m^2v^2\gamma^2$$

$$\left(\frac{E}{c}\right)^2 = m^2 \cdot (c^2 + v^2\gamma^2)$$

$$\frac{E}{c} = m \cdot \sqrt{c^2 + v^2\gamma^2}$$

$$\frac{E}{c} = m \cdot \sqrt{c^2 + v^2 \frac{1}{1-\frac{v^2}{c^2}}} = m\sqrt{c^2 + \frac{v^2c^2}{c^2-v^2}}$$

$$\frac{E}{c} = m\sqrt{\frac{c^4-c^2v^2+c^2v^2}{c^2-v^2}} = m\frac{c^2}{\sqrt{c^2-v^2}} = m\frac{c^2}{c\sqrt{1-\frac{v^2}{c^2}}} = c \cdot \gamma \text{ and consequently}$$

$$\boldsymbol{E = mc^2\gamma = m'c^2}$$

as given above in (4).

The fact that momentum has disappeared from the calculation here, in my mind, clearly explains that none can exist by assuming $m = 0$.

For a particle without rest mass is $m = 0$. Therefore, $m' = 0$ as well. This means that a particle without rest mass has neither energy nor momentum in any inertial frame of reference. Consequently, the photon, which has an experimentally verified momentum greater than zero, must have a rest mass.

Equation (12), which assumes a non-existent rest mass, cannot therefore be used in the given way to establish momentum.

I believe the cause of this misconception lies in the fundamental assumption that mass is not quantized. If one assumes that mass is quantized, and I have calculated its quantum to be $7{,}37249732 \cdot 10^{-51} kg \cdot s$, these errors can be corrected.